B.Sc.- I (CBCS Pattern) Semester-I

# USMT-01 - Mathematics Paper-I - Differential and Integral Calculus

P. Pages : 2 **GUG/S/25/11556**

Time : Three Hours * 1 5 0 6 * Max. Marks : 60

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Notes : 1. Solve all five questions.
2. Each questions carries equal marks.

## UNIT – I

**1.** a) Let $f(x)$ and $g(x)$ be defined at all points of an interval [a, b] except possibly at $x_0 \in [a,b]$. If $\lim_{x\to x_0} f(x) = A$, $\lim_{x\to x_0} g(x) = B$ then prove that- $\lim_{x\to x_0}\{f(x)+g(x)\} = \lim_{x\to x_0} f(x) + \lim_{x\to x_0} g(x) = A + B$. **6**

b) Show that the function f defined by $f(x) = x, \sin\frac{1}{x}, x \neq 0$ $= 0$ , otherwise is continuous at x = 0. **6**

**OR**

c) If f(x) is differentiable at $x = x_0$ then prove that it is continuous at $x = x_0$. **6**

d) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$ then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$. **6**

## UNIT – II

**2.** a) Prove that if a real function f defined on [a, b] is i) continuous in [a, b], ii) differentiable in (a, b) then there is at least one value $c \in (a,b)$ such that $f'(c).(b-a) = f(b) - f(a)$. **6**

b) Verify Cauchy mean value theorem for $f(x) = x^2, g(x) = x^3$ in $[1,3]$. **6**

**OR**

c) Obtain Maclaurin's series for f(x) = log (1 + x). **6**

d) Expand $f(x) = 2x^3 + 7x^2 + x - 1$ in powers of (x – 2). **6**

## UNIT – III

**3.** a) Prove that $\lceil n+1 = n\lceil n$. **6**

b) Prove that $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{\pi}{\sqrt{2}}$ **6**

**OR**

c) Prove that $\lim_{x\to 0} \log_{\tan x} \tan 2x = 1$. **6**

d) Prove that $\lim_{x \to x_0} \left( \cot x - \frac{1}{x} \right) = 0$. **6**

## UNIT – IV

**4.** a) If $f(x, y)$ and $g(x, y)$ are continuous on region D then prove that- **6**

i) $f(x, y) \geq 0 \Rightarrow \iint_D f(x, y)\, dA \geq 0$ on D.

ii) $f(x, y) \leq g(x, y) \Rightarrow \iint_D f(x, y) dA \leq \iint_D g(x, y) dA$.

b) Evaluate $\int_{-2}^{2} dx \int_{y^2-1}^{3} (x + 2y) dy$. **6**

**OR**

c) Evaluate $\iint xy(x + y) dxdy$ over the area between $y = x^2, y = x$. **6**

d) Evaluate $\iint r \sin\theta \, dr d\theta$ over the area of the cardiod r = a(1 – $\cos\theta$) above the initial line. **6**

**5.** Solve **any six** of the following.

a) Evaluate $\lim_{x \to 3} \left( 2x^3 - 3x^2 + 7x - 11 \right)$. **2**

b) Find $y_3$, if $f(x) = (2x - 3)^4$. **2**

c) State the Rolle's theorem. **2**

d) State the Taylor's theorem. **2**

e) Evaluate $\int_0^{\infty} x^3 e^{-2x} dx$. **2**

f) Prove that $B(m, n) = B(n, m)$. **2**

g) Prove that $\iint_D cf(x, y)\, dA = c \iint_D f(x, y) dA, c$ is a constant. **2**

h) Evaluate $\int_0^{\log 8} \int_0^{\log y} e^{x+y} dxdy$ **2**

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